Kaon Separator Cavity RF Drive Power Requirements

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Abstract: *The nominal RF drive power requirements for the superconducting kaon separator cavity are presented. These calculations can be used to initiate the design of the RF power distribution system of the kaon separator facility; however, they are not meant to be considered final values since a full analysis of microphonics and Lorentz force detuning effects still has to be carried out.*

The forward drive power required to maintain a certain amount of steady-state stored energy within a cavity which is coupled to the end of an input transmission line is given as

$$P_{FWD} = \frac{(\beta+1)^2}{4\beta} \cdot \frac{\omega_o U_{cav}}{Q_o} \qquad (1)$$

where P_{FWD} is the power associated with the forward wave on the transmission line, β is the coupling coefficient of the input coupler / cavity arrangement, ω_o is the radian resonant frequency, U_{cav} is the energy stored in the cavity in steady state, and Q_o is the unloaded quality factor of the cavity. The derivation of this equation is explained in Appendix B.

In the superconducting case, high values for Q_o are achieved. This is good in terms of cavity dissipated power but presents a problem for dynamically controlling the stored energy in the presence of disturbances since the cavity bandwidth is low. This problem is alleviated by choosing high coupling coefficients, thus achieving a low loaded quality factor.

Thus, when $\beta >> 1$, Eq. 1 can be approximated by

$$P_{FWD} \approx \frac{1}{4} \cdot \frac{\omega_o U_{cav}}{Q_L} \quad . \qquad (2)$$

where the facts[1] that $\beta = \frac{Q_o}{Q_{ext}}$ and $\left(\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_{ext}}\right)$ were used (Note: $Q_L \approx Q_{ext}$, when $\beta >> 1$). This states that in the superconducting case the forward drive power is roughly ¼ of the power that would be dissipated in a cavity with a quality factor of Q_L.

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[1] For an explanation of coupling coefficient and quality factors see E.L. Ginzton, “Microwave Measurements”, McGraw-Hill Book Co., 1957, Library of Congress Catalog Number 56-13393, pp. 391-399, or J.C. Slater, “Microwave Electronics”, D. Van Nostrand Company Inc., 1950, Chapters 3 and 4.

For the Kaon separator, at this time, it is desired to operate at a deflecting gradient of 5 MV/m. MAFIA simulations[2] predict that the ratio, κ_D, of the deflecting gradient, E_D, to the square root of the cavity energy for a 13-cell cavity structure of shape C15 is

$$\kappa_D = \frac{E_D}{\sqrt{U_{cav}}} = 8.283 \frac{MV/m}{\sqrt{J}} \quad .$$

Using this value, the steady-state stored cavity energy for a 5 MV/m deflecting gradient is 0.364 J. The design specification at this time dictates a loaded quality factor, Q_L, of $6 \cdot 10^7$. The operating frequency of the structure is 3.9GHz. Using these numbers in Eq. 2 , the forward drive power requirement for one 13-cell cavity structure is roughly 37 Watts. This does not include any power distribution losses.

Detailed Mathcad[3] calculations for the entire Kaon Separator 13-cell cavity facility using the exact expression in Eq. 1 and not including power distribution losses can be found in Appendix A. The results are summarized below.

Kaon Separator Design Specifications and RF Power Requirements

Frequency	3.9	GHz
Q_o	$2.1 \cdot 10^9$	
Q_L	$6 \cdot 10^7$	
E_D (Deflecting Gradient)	5	MV/m
U_{cav} (13-cell stored Energy)	0.364	J
κ_D	8.283	$\frac{MV/m}{\sqrt{J}}$
P_{Cav} (Cavity Dissipated Power)	4.25	Watts
$P_{FWD\ at\ Cavity}$	38.3	Watts
Number of Cavities per Cryo	2	
P_{FWD} per Cryostat	76.6	Watts
Cryostat effective RF Length	1	m
Total effective RF Length	6	m
Total number of 13-cell cavities	12	
Design Margin	2	
Total RF Source Power Required	920	Watts

The above numbers do not include losses associated with the power distribution system nor do they include excessive power needed for overcoming disturbances such as microphonics or Lorentz force detuning. However, a design margin of 2 was included. Beam loading should be negligible since this is a deflecting mode cavity. The deflection is achieved mainly from the magnetic field with little contribution from the electric field of the cavity. Thus, very little energy is imparted to the beam.

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[2] M. McAshan and R. Wanzenberg, “RF Design of a Transverse Mode Cavity for Kaon Separation”, Fermilab TM-2144, Batavia, 2001.
[3] Mathcad is written by Mathsoft, Inc. 101 Main Street, Cambridge, MA 02142

Source Power Requirements for the A0 Kaon Separator

Definitions:

$MV := 1000KV \quad mT := 1 \cdot 10^{-3} T \quad c := 3 \cdot 10^{8} \frac{m}{s} \quad i := i$

13-Cell Cavity Parameters:

$f := 3.9 \cdot GHz \quad \omega := 2 \cdot \pi \cdot f \quad \lambda := \frac{c}{f} \quad \lambda = 0.077\,m$

$Q_o := 2.1 \cdot 10^{9} \quad Q_L := 6 \cdot 10^{7} \quad Q_{ext} := \frac{Q_o \cdot Q_L}{Q_o - Q_L} \quad Q_{ext} = 6.18 \times 10^{7}$

$\beta := \frac{Q_o}{Q_{ext}} \quad \beta = 34$

$\kappa_E := 30.65 \frac{MV}{m \cdot \sqrt{J}}$ — κ_E is the ratio of peak E-field to the sqrt of Cavity Energy.

$\kappa_D := 8.283 \frac{MV}{m \cdot \sqrt{J}}$ — κ_D is the ratio of deflecting gradient to the sqrt of Cavity Energy.

$BpEd := 15.40 \frac{mT}{\left(\frac{MV}{m}\right)}$ — BpEd is the ratio of peak magnetic field to deflecting gradient.

$EpEd := 3.70$ — EpEd is the ratio of peak electric field to deflecting gradient.

$Ed_{operate} := 5 \frac{MV}{m}$ — $Ed_{operate}$ is the operating deflecting gradient.

Source Power Calculations Per 13-Cell Cavity:

$$U_{cav} := \frac{Ed_{operate}^{2}}{\kappa_D^{2}} \qquad U_{cav} = 0.364\,\mathrm{J}$$

$$P_{cav} := \frac{\omega \cdot U_{cav}}{Q_o} \qquad P_{cav} = 4.25\,\mathrm{W}$$

$$\Gamma_{cav} := \frac{(\beta - 1)}{(\beta + 1)} \qquad \Gamma_{cav} = 0.943$$

$$P_{source} := \frac{P_{cav}}{1 - \left(\left|\Gamma_{cav}\right|\right)^2} \qquad P_{source} = 38.3\,\mathrm{W}$$

Comparison to Calculation from Q_L

$$P_{QL} := \frac{\omega \cdot U_{cav}}{Q_L} \qquad P_{QL} = 148.8\,\mathrm{W}$$

$$\frac{P_{QL}}{P_{source}} = 3.89$$

Source Power Calculations for Entire Facility:

$num\_cav_{per\_cryo} := 2$

$P_{per\_cryostat} := num\_cav_{per\_cryo} \cdot P_{source}$ $\qquad P_{per\_cryostat} = 76.6\,\mathrm{W}$

$num\_cryos_{per\_station} := 3$

$P_{per\_station} := num\_cryos_{per\_station} \cdot P_{per\_cryostat}$ $\qquad P_{per\_station} = 229.8\,\mathrm{W}$

$num_{stations} := 2$

$safety_{factor} := 2$

$P_{RF\_Total} := safety_{factor} \cdot num_{stations} \cdot P_{per\_station}$ $\qquad P_{RF\_Total} = 919.2\,\mathrm{W}$

Total RF Source Power Requirements for Entire Facility including safety factor.

Note: This does not include Power Distribution Losses

$$P_{RF\_Total} = 919.2\,\mathrm{W}$$

Appendix B – Derivation of the Drive Power Requirement Equations

An equivalent circuit model[1] representing a driven cavity is shown in Figure 1. The transformer models the coupling of the cavity to the input transmission line via the input coupler.

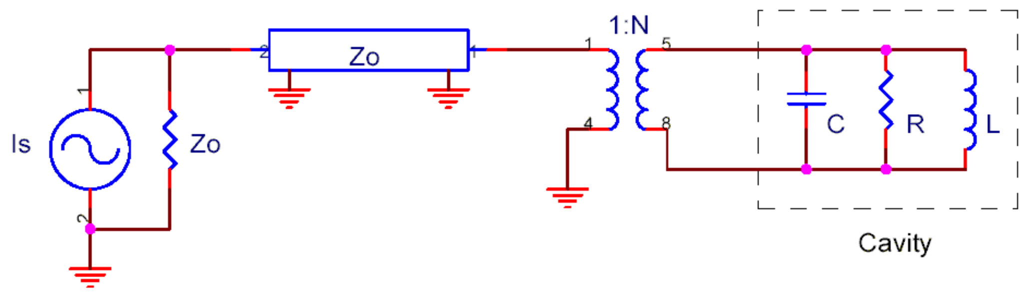

Figure 1: Cavity Coupled to a Transmission Line

The cavity impedance can be written as:

$$Z_{cav} = \frac{R}{1 + i\,Q_o\left(\frac{\omega^2 - \omega_o{}^2}{\omega\omega_o}\right)} \quad ,$$

where R is the equivalent shunt impedance of the cavity, Q_o, is the unloaded quality factor, ω is the drive frequency, and ω_o is the resonant frequency of the cavity.

The coupling coefficient, β, is defined[1] as,

$$\beta = \frac{R}{N^2 Z_o} = \frac{Q_o}{Q_{ext}} \quad ,$$

where N is the transformer turns ratio, Z_o is the characteristic impedance of the line, and Q_{ext} is the external quality factor. The impedance which the cavity presents to the input transmission line can then be expressed as,

[1] This model is commonly used to describe a cavity coupled to a transmission line. A discussion of this model can be found in E.L. Ginzton, “Microwave Measurements”, McGraw-Hill Book Co., 1957, Library of Congress Catalog Number 56-13393, pp. 391-397. For the development of the model see C.G. Montgomery, R.H. Dicke, E.M. Purcell, “Principles of Microwave Circuits”, McGraw-Hill Book Co., 1948, Chapter 7.

$$Z_L = \frac{\beta Z_o}{1 + i\,Q_o\left(\dfrac{\omega^2 - \omega_o^{\,2}}{\omega\omega_o}\right)} \quad .$$

On resonance ($\omega = \omega_o$) , the load impedance presented to the transmission line is simply βZ_o . Thus, the reflection coefficient[2] on resonance at the cavity end of the input transmission line can be expressed as

$$\Gamma_L = \frac{\beta - 1}{\beta + 1} \quad .$$

The power which is delivered to the cavity is simply given as

$$P_{cav} = P_{FWD} \cdot \left(1 - \left|\Gamma_L\right|^2\right) \ , \text{ or}$$

$$P_{cav} = \frac{4\beta}{\left(\beta + 1\right)^2} \cdot P_{FWD}$$

The cavity power can also be expressed in terms of the stored cavity energy, U_{cav} , and the unloaded quality factor, Q_o , of the cavity,

$$P_{cav} = \frac{\omega_o U_{cav}}{Q_o}$$

Thus, for a desired cavity energy, the forward power requirement is given as

$$P_{FWD} = \frac{\left(\beta + 1\right)^2}{4\beta} \cdot \frac{\omega_o U_{cav}}{Q_o}$$

In the superconducting case when $\beta \gg 1$ and $Q_L \approx Q_{ext}$, the required forward power is

$$P_{FWD} = \frac{1}{4} \cdot \frac{\omega_o U_{cav}}{\left(\dfrac{Q_o}{\beta}\right)} = \frac{1}{4} \cdot \frac{\omega_o U_{cav}}{Q_{ext}} \approx \frac{1}{4} \cdot \frac{\omega_o U_{cav}}{Q_L} \quad .$$

Thus, the required forward power is ¼ the value of the power that would be dissipated in a cavity with a quality factor equal to Q_L .

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[2] For a discussion of reflection coefficient and transmission line concepts see G. Gonzalez, “Microwave Transistor Amplifiers”, Prentice-Hall, Engelwood Cliffs, 1984, Chapter 1.